

# Memo : Specifications for reference frame fixing in the analysis of a EUREF GPS campaign

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## 1. Introduction

The goal is to process GPS data in the commonly adopted ETRS89 system and taking full benefit of most recent fiducials or GPS ephemerides as provided by IGS.

Basic principles has been agreed by the TWG to define the procedure described below. They can be summarized according to this way:

1. to take full benefit of the successively improved realizations of the IERS Terrestrial Reference System (ITRS), known as  $ITRF_{YY}$  (published in the IERS Annual Report for  $YY$ ). This realization consists into a list of points (station references or markers) together with:

- positions at epoch  $t_0$ ,  $X_{YY}^I(t_0)$
- velocities  $\dot{X}_{YY}^I$

so that the position of a point at epoch  $t$  will be :

$$X_{YY}^I(t) = X_{YY}^I(t_0) + \dot{X}_{YY}^I \cdot (t - t_0)$$

2. to accept that the general model for transformation from a system A to a system B will be:

$$\begin{pmatrix} X_B \\ Y_B \\ Z_B \end{pmatrix} = \begin{pmatrix} X_A \\ Y_A \\ Z_A \end{pmatrix} + \begin{pmatrix} T1_{A,B} \\ T2_{A,B} \\ T3_{A,B} \end{pmatrix} + \begin{pmatrix} D_{A,B} & -R3_{A,B} & R2_{A,B} \\ R3_{A,B} & D_{A,B} & -R1_{A,B} \\ -R2_{A,B} & R1_{A,B} & D_{A,B} \end{pmatrix} \begin{pmatrix} X_A \\ Y_A \\ Z_A \end{pmatrix}$$

where the transformation parameters can be linearly dependent of time. So, for a transformation parameter P, we have: .

$$P_{A,B}(t) = P_{A,B}(t_0) + \dot{P}_{A,B} \times (t - t_0)$$

3. to accept that any new frame validated by the TWG would have minimum systematic shift with regard to the EUREF89 frame, but would stick to its own scale especially if it is significantly more accurate than the scale underlying EUREF89.

In addition to these principles, the fullfilment of the Bern Resolution concerning ETRS89 should be clearly realized.

## 2. Specifications for realizations derived from ITRF

As previously described (Boucher and Altamimi, 1992), one can derive from each annual frame determined by IERS under the label  $ITRF_{YY}$ , a corresponding frame in ETRS89, which will be itself labelled  $ETRF_{YY}$ .

The detailed specifications to establish  $ETRF_{YY}$  are:

### 1. Selection of points

All points corresponding to sites belonging to ITRF and located in Europe (nominally up to Ural) will be selected.

Occasionally additional markers or points can be added (RETRIG markers, new GPS tracking, other systems such as DORIS or PRARE...) if local eccentricities are available between it and some point already existing in ITRF.

### 2. Coordinates and velocities

These values are obtained as the following:

- (a) compute at 89.0 in ITRS

$$X_{YY}^I(89.0) = X_{YY}^I(t_0) + \dot{X}_{YY}^I \times (89.0 - t_0)$$

- (b) compute in ETRS at 89.0:

$$\begin{pmatrix} X_{YY}^E(89.0) \\ Y_{YY}^E(89.0) \\ Z_{YY}^E(89.0) \end{pmatrix} = \begin{pmatrix} X_{YY}^I(89.0) \\ Y_{YY}^I(89.0) \\ Z_{YY}^I(89.0) \end{pmatrix} + \begin{pmatrix} T1_{YY} \\ T2_{YY} \\ T3_{YY} \end{pmatrix}$$

where  $T_{YY}$  is given in Appendix 1.

- (c) compute velocity in ETRS:

$$\begin{pmatrix} \dot{X}_{YY}^E \\ \dot{Y}_{YY}^E \\ \dot{Z}_{YY}^E \end{pmatrix} = \begin{pmatrix} \dot{X}_{YY}^I \\ \dot{Y}_{YY}^I \\ \dot{Z}_{YY}^I \end{pmatrix} + \begin{pmatrix} 0 & -\dot{R}3_{YY} & \dot{R}2_{YY} \\ \dot{R}3_{YY} & 0 & -\dot{R}1_{YY} \\ -\dot{R}2_{YY} & \dot{R}1_{YY} & 0 \end{pmatrix} \times \begin{pmatrix} X_{YY}^I \\ Y_{YY}^I \\ Z_{YY}^I \end{pmatrix}$$

where  $\dot{R}_{YY}$  is given in Appendix 2.

### 3. Specifications to compute a EUREF GPS campaign in ETRS 89

Given a set of GPS measurements referred to a central epoch  $t_c$ , the procedure will be:

1. to process data in ITRS at epoch  $t_c$

For that purpose, use recent  $ITRF_{YY}$ . If IGS ephemerides are used, take the  $YY$  corresponding to the one used by IGS to generate the ephemerides.

The stations used for GPS tracking during this campaign and for which accurate (cm level) coordinates are available in  $ITRF_{YY}$  will be held fixed (or strongly constrained) to the values:

$$X_{YY}^I(t_c) = X_{YY}^I(t_0) + \dot{X}_{YY}^I \times (t_c - t_0)$$

The results are then all consistent with  $ITRF_{YY}$  at epoch  $t_c$ .

2. convert in ETRS89 at  $t_c$  :

$$X^E(t_c) = X_{YY}^I(t_c) + T_{YY} + \begin{pmatrix} 0 & -\dot{R}_{3YY} & \dot{R}_{2YY} \\ \dot{R}_{3YY} & 0 & -\dot{R}_{1YY} \\ -\dot{R}_{2YY} & \dot{R}_{1YY} & 0 \end{pmatrix} \times X_{YY}^I(t_c) \cdot (t_c - 1989.0)$$

where  $T_{YY}$  is given in Appendix 1 and  $\dot{R}_{YY}$  in Appendix 2.

3. to express at 89.0 :

$$X^E(89.0) = X^E(t_c) + \dot{X}^E \cdot (1989.0 - t_c)$$

where  $\dot{X}^E$  is an estimation of the velocity of the station in ETRS. For stable part, one may use  $\dot{X}^E = 0$ .

### 4. Appendix 1: Estimation of shift $T_{YY}$

Two solutions are available:

**A)** use estimated global offsets between successive  $ITRF_{YY}$ . Table 1 gives the parameters from  $YY$  to 89 at epoch  $t_0$ , and Table 2 their secular changes.

If we define  $\bar{X}$  as the barycenter of the ETRF89 network, then the transformation parameters at 89.0 are:

$$T_{YY,89} = T_{YY,89}(t_0) + \dot{T}_{YY,89} \times (89.0 - t_0)$$

$$D_{YY,89} = D_{YY,89}(t_0) + \dot{D}_{YY,89} \times (89.0 - t_0)$$

$$R_{YY,89} = R_{YY,89}(t_0) + \dot{R}_{YY,89} \times (89.0 - t_0)$$

and the equivalent shift is:

$$T_{YY} = T_{YY,89} + \begin{pmatrix} D_{YY,89} & -R3_{YY,89} & R2_{YY,89} \\ R3_{YY,89} & D_{YY,89} & -R1_{YY,89} \\ -R2_{YY,89} & R1_{YY,89} & D_{YY,89} \end{pmatrix} \bar{X}$$

**B)** compute shift on ETRF89 stations. Compute  $T_{YY}$  by a 3 parameters fit between  $X_{89}^E(89.0)$  (or EUREF 89 values) and  $X_{YY}^I(89.0)$

Table 3 gives the estimations of  $T_{YY}$  according to A and B. Since the two estimations are equivalent regarding the error bars, we recommend the use of case A values.

**Table 1:** Transformation parameters from  $ITRF_{YY}$  to ITRF89

From	T1 cm	T2 cm	T3 cm	D $10^{-8}$	R1 mas	R2 mas	R3 mas	$t_0$ y	Ref. IERS TN
ITRF90	0.5	2.4	-3.8	0.34	0.0	0.0	0.0	88.0	9
ITRF91	0.6	2.0	-5.4	0.37	0.0	0.0	0.0	88.0	12
ITRF92	1.7	3.4	-6.0	0.51	0.0	0.0	0.0	88.0	15
ITRF93	1.9	4.1	-5.3	0.39	0.39	-0.80	0.96	88.0	18
ITRF94	2.3	3.6	-6.8	0.43	0.0	0.0	0.0	88.0	21
ITRF96	2.3	3.6	-6.8	0.43	0.0	0.0	0.0	88.0	24
ITRF97	2.3	3.6	-6.8	0.43	0.0	0.0	0.0	88.0	27
ITRF2000	3.0	4.2	-8.7	0.59	0.0	0.0	0.0	97.0	
ITRF2005	3.0	3.9	-9.7	0.63	0.0	0.0	0.06	00.0	

**Table 2:** Rates of change of the transformation parameters from  $ITRF_{YY}$  to ITRF89

From	$\dot{T}1$ cm/y	$\dot{T}2$ cm/y	$\dot{T}3$ cm/y	$\dot{D}$ $10^{-8}/y$	$\dot{R}1$ mas/y	$\dot{R}2$ mas/y	$\dot{R}3$ mas/y	Ref. IERS TN
ITRF90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
ITRF91	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
ITRF92	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
ITRF93	0.29	-0.04	-0.08	0.0	0.11	0.19	-0.05	18
ITRF94	0.0	0.0	0.0	0.0	0.0	0.0	0.0	21
ITRF96	0.0	0.0	0.0	0.0	0.0	0.0	0.0	24
ITRF97	0.0	0.0	0.0	0.0	0.0	0.0	0.0	27
ITRF2000	0.0	-0.06	-0.14	0.0	0.0	0.0	0.02	
ITRF2005	-0.02	-0.05	-0.32	0.008	0.0	0.0	0.02	

**Table 3:** Estimation of  $T_{YY}$ 

YY		T1 cm	T2 cm	T3 cm
89		0	0	0
90	A	1.9	2.8	-2.3
	B $\pm$	2.6 0.7	2.5 0.7	-2.6 0.7
91	A	2.1	2.5	-3.7
	B $\pm$	2.3 0.7	2.1 0.7	-3.1 0.7
92	A	3.8	4.0	-3.7
	B $\pm$	4.3 0.8	3.4 0.8	-3.2 0.8
93	A	1.9	5.3	-2.1
	B $\pm$	1.0 0.5	5.9 0.5	-1.4 0.6
94	A	4.1	4.1	-4.9
	B $\pm$	2.9 0.4	4.3 0.5	-3.6 0.5
96	A	4.1	4.1	-4.9
	B $\pm$	3.9 0.4	4.1 0.4	-3.9 0.4
97	A	4.1	4.1	-4.9
	B $\pm$	3.4 0.4	4.4 0.4	-4.3 0.4
00	A	5.4	5.1	-4.8
	B $\pm$	4.2 0.4	5.1 0.4	-4.6 0.4
05	A	5.6	4.8	-3.7
	B $\pm$	3.6 0.4	4.2 0.4	-4.1 0.4

## 5. Appendix 2: Estimation of $\dot{R}_{YY}$

Since the associated velocity fields of ITRF89 and ITRF90 are computed using AM0-2 model (Minster and Jordan, 1978),  $\dot{R}_{YY}$  will be the angular velocity of the Eurasian plate in this model.

On the other hand there are two estimated velocity fields associated with ITRF91 and ITRF92 respectively. In these two frames, the orientation time evolution was ensured by aligning the corresponding velocity fields to NNR-NUVEL-1 model (Argus et Gordon, 1991, De Mets et al, 1990). So for 91 and 92,  $\dot{R}_{YY}$  corresponds, conventionally, to the angular velocity of the Eurasian plate in NNR-NUVEL-1 model.

The more recent geophysical model NNR-NUVEL-1A (DeMets et al, 1994) has been used as reference in the ITRF93 velocity field computation. It should be noted that there is a rotation rate between the ITRF93 velocity field and the NNR-NUVEL-1A model (Boucher et al, 1994). Consequently for 93,  $\dot{R}_{YY}$  corresponds to the angular velocity of the Eurasian plate in NNR-NUVEL-1A model to which we added the rotation rate between the ITRF93 velocity field and the NNR-NUVEL-1A model.

As the time evolution of the ITRF94 is consistent with the model NNR-NUVEL-1A (Boucher et al, 1996), then the  $\dot{R}_{YY}$  corresponds, conventionally, to the angular velocity of the Eurasian plate in this model.

The reference frame definition (origin, scale, orientation and time evolution) of the ITRF96 is achieved in such a way that ITRF96 is in the same system as ITRF94 (Boucher et al, 1998). Consequently,  $\dot{R}_{YY}$  is the same as for ITRF94. This same statement is also valid for ITRF97.

For the first time, the ITRF2000 combines individual solutions that are free from any plate motion model. Its origin is defined by a weighted average of most consistent SLR solutions. Its scale is defined by most consistent SLR and VLBI solutions. Its orientations is aligned to the ITRF97 at epoch 1997.0 and its orientation rate follows, conventionally, that of NNR-NUVEL-1A model. The ITRF2000 velocity field was used to estimate angular velocities of 6 major plates, including Eurasia, showing significant disagreement with NUVEL-1A predictions. It is therefore recommended to use for  $\dot{R}_{YY}$  the components of the Eurasian angular velocity estimated from ITRF2000 velocities of 19 European sites of high geodetic quality. For more details, see (Altamimi et al., 2002). Using a velocity field of 152 sites of high quality extracted from the ITRF2005 solution (Altamimi et al., 2007), absolute rotation poles of 15 tectonic plates (including Eurasia) were estimated. The components of the Eurasia plate rotation pole are those corresponding to the values of  $\dot{R}_{YY}$  listed in Table 4 to be used in the transformation from ITRF2005 to ETRF2005.

**Table 4:** Estimation of  $\dot{R}_{YY}$

YY	$\dot{R}1$ mas/y	$\dot{R}2$ mas/y	$\dot{R}3$ mas/y
89	0.11	0.57	-0.71
90	0.11	0.57	-0.71
91	0.21	0.52	-0.68
92	0.21	0.52	-0.68
93	0.32	0.78	-0.67
94	0.20	0.50	-0.65
96	0.20	0.50	-0.65
97	0.20	0.50	-0.65
00	0.081	0.490	-0.792
	$\pm 0.021$	$\pm 0.008$	$\pm 0.026$
05	0.054	0.518	-0.781
	$\pm 0.009$	$\pm 0.006$	$\pm 0.011$

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